Prof. Ha

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THE

MATHEMATICAL MONTHLY.

APRIL, 1861.

EDITED BY

J. D. RUNKLE, A.M., A.A.S.

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THE MATHEMATICAL MONTHLY.

The excellent notices which the Monthly continues to receive from both sides of the Atlantic must be a source of great encouragement to all its friends. The *Lady's and Gentleman's Diary* for 1861, edited by the eminent mathematician, PROF. W. S. B. WOOLHOUSE, contains the following: —

"This American periodical, which has now completed its second volume, continues to be most ably conducted, and is well supplied by talented correspondents. Besides numerous mathematical questions and solutions, it comprises original papers of considerable variety and sterling interest, and its publication must contribute largely to the diffusion and advancement of mathematical science."

A recent number of the *Boston Daily Transcript* gives the following item: —

"AMERICAN LITERATURE ABROAD. — Fourteen columns of the last Athenæum are devoted to two American authors, — Motley and Emerson. That paper, which rarely commends anything from this side of the Atlantic, gives 'The Mathematical Monthly' the following first-rate notice. That magazine is published at Cambridge, and edited by Mr. J. D. Runkle: —

"'This is a collection of simple things for comparative elementary students. Our English mathematical journals are very lofty, and the beginner has no part in them. This American journal is full of pleasing elementary matter, and contains some things worthy the attention of the finished mathematician. We should like to see the same sort of thing in England; but where are we to find the editor? He must be of high knowledge and elementary turn combined, with firmness enough to keep his journal down to the intelligible point by refusal of contributions of too lofty a character, and skill enough to keep it up to the interesting point by selecting from the loads of lower learning with which he would be inundated.'"

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The Mathematical Monthly.

IN commencing our third volume, we beg to assure the friends of this Journal, that all legitimate efforts will be made to sustain it, together with the Prizes, which have proved such an excellent feature of its plan. It will be seen (see 2d page of cover) that the amount of the Prizes has been increased. But all efforts on our part must prove unavailing if the subscription list, with other sources of income, will not cover the expenses which we are incurring to make the Monthly really valuable and attractive to all interested in it. To show our friends at home what is thought of the Monthly abroad, we venture to make short extracts from letters lately received

"The work appears to me so excellent, that I have at once ordered the series for the Library of the Observatory.

"G. B. AIRY, *Ast. Royal.*"

"*Royal Obs., Greenwich, July* 5 1860.

"I have looked through it with great interest, and especially because it shows to what an extent interest in mathematical subjects is felt through the United States. I do not believe that it would be possible in England to get up a Mathematical periodical brought out in so handsome a form and supported by so large a number of subscribers.

"*Trinity College, Dublin, July* 12, 1860. GEORGE SALMON."

"DEAR SIR, — I thank you sincerely for the copy you so kindly sent me of the first volume of the Mathematical Monthly We have not, that I am aware of, any work of a similar character in England. I feel confident it will serve a valuable purpose in stimulating and giving a healthy tone to the study of Mathematics in the higher class of schools, and judging from this volume, it will be a work of as great interest and give as much matter for thought to the tutor as to the student Original investigations on the mere elementary branches of science are perhaps not often to be expected: those portions have now been so long and so frequently considered, and by so many different minds, that something new — a really original idea — appears only at very distant intervals. Still many interesting questions may, and, as your first volume shows, do arise even in the earliest branches of Mathematical science; whether discussions on the principles and on the best methods of placing them before the pupil, or examples and problems which in their solutions show a large amount of ingenuity and mathematical skill. The Mathematical Monthly claims a special interest from the very fact of its character rendering it useful to the many without detracting from its scientific merit.

"I remain, your obedient servant.

"*St. John's College, Cambridge, July* 26, 1860. HUGH GODFRAY."

We feel confident that a little effort on the part of our friends would double our subscription list. See terms to clubs and new subscribers taking Vols. I., II., III. We could then engrave some of the fine portraits of LAGRANGE, MONGE, CAUCHY, BIOT, &c., lately received; and, indeed, incur any expense to increase the value of the Monthly to all Will our friends make up their clubs at once, and notify us, so that we can decide upon the size of our edition?

J F W. Herschel

THE

MATHEMATICAL MONTHLY.

Vol. III. . . . APRIL, 1861. . . . No. VII.

PRIZE PROBLEMS FOR STUDENTS.

I. Let ACB, ADB be two right-angled triangles having a common hypothenuse, AB; join CD, and on CD produced both ways draw perpendiculars AE, BF. Show that

$$CE^2 + CF^2 = DE^2 + DF^2.$$

II. A clock gains $3\frac{1}{4}$ minutes in 15 seconds under the 24 hours. At noon it is two minutes slow; when will it indicate correct time?

III. Two railroads intersect at an angle φ; and on these roads, distant from the junction a and b miles respectively, are two trains running towards the junction at the rates of m and n miles respectively. When and where will these trains be at a minimum distance from each other? — Communicated by Prof. Wray Beattie, Wesleyan University, Mt. Pleasant, Iowa.

IV. The former of the two similar triangles $A'B'C'$ and ABC is circumscribed about the triangle DEF, and the latter is inscribed in DEF. Show that DEF is a mean proportional between $A'B'C'$ and ABC only when the homologous sides of the latter are parallel; also determine when ABC is a minimum, $A'B'C'$ and DEF being given. — Communicated by Asher B. Evans, Madison University, Hamilton, N. Y.

V. If r be the radius of the sphere inscribed in a tetrahedron formed by three rectangular co-ordinate planes and the plane

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, and d be the distance of this plane from the origin, prove that $\frac{1}{r} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$.

Solutions of these problems must be received by June 1st, 1861.

AWARD OF THE PRIZES FOR SOLUTIONS OF PROBLEMS IN No. III., Vol. III.

THE first prize is awarded to GEORGE B. HICKS, Cleveland, Ohio.

The second prize is awarded to ASHER B. EVANS, Madison University, Hamilton, N. Y.

The third prize is awarded to DAVID TROWBRIDGE, Perry City, N. Y.

The fourth prize is awarded to W. W. LEMMON, M'Kendree College, Lebanon, Illinois.

PRIZE SOLUTION OF PROBLEM I.

By W. W. LEMMON, M'Kendree College, Lebanon, Ill.

From AC, the diagonal of a square, $ABCD$, cut off AE equal to one fourth of AC, and join BE, DE. Show that the figure $BADE$ equals twice the square on AE.

The triangles ADE and ABE are equal, because they have the same base, AE, and the equal altitudes; and each is equal to one fourth of ADC or ABC. Therefore $4\,BADE = AB^2$; and since $AC^2 = 2\,AB^2$, we have

$$BADE = \tfrac{1}{4}\,AB^2 = \tfrac{1}{8}\,AC^2 = \tfrac{1}{8}\,(4\,AE)^2 = 2\,AE^2.$$

Mr. WILLIAM TIMPSON says: Since the semi-diagonal AF is bisected in E, EG parallel to FB bisects AB in G. The triangle

$$AEG = \tfrac{1}{2}\,AE^2 = GEB\,;$$

hence $AEB = AE^2$; $\therefore BADE = 2\,AE^2$. Or, complete the square $FEGH$. Draw BE, cutting GH in L, and produce it to meet DA in M. The triangles EGL, BHL being equal, the triangle

$$BFE = BEA = EF^2 = AE^2,$$

as before.

Corollary. — The triangles AME, BCE being similar,

$$(EC = 3\,AE) : AE = EB : EM = (BC = AD) : AM;$$

or $$3 : 1 = EB : EM = AD : AM; \quad \therefore AM = \tfrac{1}{3}\,AD;$$

and $$4 : 1 = (EB + EM = BM) : EM; \quad \therefore EM = \tfrac{1}{4}\,BM.$$

Prize Solution of Problem II.

By E. S. Stearns, Chester, Morris Co., N. J.

Two toothed wheels work against each other. Show that, if the number of teeth in one be prime to that in the other, before two teeth, which have once been in contact, come in contact again, every tooth of the one wheel will have been in contact with every tooth of the other.

Let $a > b$, prime to each other, be the number of teeth in the wheels A and B; then, before any two teeth which are together can come together again, each wheel must make a whole number of revolutions; A makes b, and B makes a revolutions. Again, when B makes one revolution, every tooth of B has been in contact with one tooth of A, and in making a revolutions every tooth of B has been in contact with a, or every tooth of A.

Prize Solution of Problem III.

By George B. Hicks, Cleveland, Ohio.

Eliminate φ from the equations

$$x \cos(\varphi + \alpha) + y \sin(\varphi + \alpha) = a \sin 2\varphi,$$
$$y \cos(\varphi + \alpha) - x \sin(\varphi + \alpha) = 2\,a \cos 2\varphi,$$

showing that

$$(x \sin\alpha - y\cos\alpha)^{\frac{2}{3}} + (y \sin\alpha + x\cos\alpha)^{\frac{2}{3}} = (2\,a)^{\frac{2}{3}}.$$

Develop the left-hand members of these equations, and divide the first by the second; we get

$$\frac{x\cos\alpha + y\sin\alpha + \tan\varphi\,(y\cos\alpha - x\sin\alpha)}{y\cos\alpha - x\sin\alpha - \tan\varphi\,(x\cos\alpha + y\sin\alpha)} = \frac{\tan 2\varphi}{2} = \frac{\tan\varphi}{1 - \tan^2\varphi}.$$

$$\therefore \tan^3\varphi = \frac{x\cos\alpha + y\sin\alpha}{y\cos\alpha - x\sin\alpha} = \frac{m}{n}.$$

$$\therefore \sin \varphi = \frac{m^{\frac{1}{3}}}{(m^{\frac{2}{3}} + n^{\frac{2}{3}})^{\frac{1}{2}}}, \qquad \cos \varphi = \frac{n^{\frac{1}{3}}}{(m^{\frac{2}{3}} + n^{\frac{2}{3}})^{\frac{1}{2}}}.$$

But the first equation may be written

$$\frac{m}{\sin \varphi} + \frac{n}{\cos \varphi} = 2\,a = (m^{\frac{2}{3}} + n^{\frac{2}{3}})^{\frac{3}{2}}.$$

Hence, observing that $n^{\frac{2}{3}}$ and $(-n^{\frac{2}{3}})$ are identical, we obtain

$$m^{\frac{2}{3}} + n^{\frac{2}{3}} = (x \sin \alpha - y \cos \alpha)^{\frac{2}{3}} + (y \sin \alpha + x \cos \alpha)^{\frac{2}{3}} = (2\,a)^{\frac{2}{3}}.$$

This elegant solution is the only complete one received, owing doubtless to the misprint of a sign in the problem.

Prize Solution of Problem IV.

By Asher B. Evans, Madison University, Hamilton, N. Y.

Show that $\frac{1}{4}\,m^2\sqrt{3}$ is the area of the greatest triangle which can be formed with the lines a, b, c, subject to the condition $a^3 + b^3 + c^3 = 3\,m^3$.

If A denotes the area of the triangle, then

$$u = 16\,A^2 = 2\,a^2 b^2 + 2\,a^2 c^2 + 2\,b^2 c^2 - a^4 - b^4 - c^4$$

must be a maximum subject to the given condition. Therefore

$$(1) \qquad \tfrac{1}{4}\,d\,u = (a\,b^2 + a\,c^2 - a^3)\,d\,a + (b\,a^2 + b\,c^2 - b^3)\,d\,b + (c\,a^2 + c\,b^2 - c^3)\,d\,c = 0,$$

$$(2) \qquad a^2\,d\,a + b^2\,d\,b + c^2\,d\,c = 0.$$

Multiply (2) by λ, an indeterminate multiplier, add the product to (1), and equate the coefficients of the arbitrary differentials $d\,a$, $d\,b$, $d\,c$, to zero, which gives

$$a\,(b^2 + c^2 - a^2 + \lambda\,a) = 0, \qquad b\,(a^2 + c^2 - b^2 + \lambda\,b) = 0,$$
$$c\,(a^2 + b^2 - c^2 + \lambda\,c) = 0.$$

In these three equations, put $b = n\,a$, $c = r\,a$, and they become

$$(n^2+r^2-1)a+\lambda=0, \qquad (1+r^2-n^2)a+\lambda n=0,$$
$$(1+n^2-r^2)a+\lambda r=0.$$

By adding the first and second, first and third, and second and third of these equations, we get

$$2r^2a+\lambda(1+n)=0, \quad 2n^2a+\lambda(1+r)=0, \quad 2a+\lambda(r+n)=0;$$

$$\therefore \lambda=-\frac{2r^2a}{1+n}=-\frac{2n^2a}{1+r}=-\frac{2a}{r+n},$$

$$\therefore r^3=n^3+n^2-r^2=n+1-r^2n; \quad \therefore \{(n+1)^2+r^2\}(n-1)=0;$$

$$\therefore n-1=0, \qquad \text{and} \qquad (n+1)^2+r^2=0.$$

Since the sum of the squares of two real quantities cannot be zero, the only real value of n is unity, which gives $r=1$. Therefore $a=b=c$, and from the given condition

$$a=b=c=m; \quad \therefore A=\tfrac{1}{4}m^2\sqrt{3}.$$

Mr. Trowbridge shows that $A=\frac{1}{4}m^2\sqrt{3}$ is also the maximum triangle when $a^n+b^n+c^n=3m^n$.

Prize Solution of Problem V.

The equation of a family of ellipses is $ax^2+by^2=1$, when $a-b=c$, a constant. Show that the curve, which cuts all the individuals at an angle whose tangent is $\frac{y}{x}$, is $x=Ce^{\frac{cy^2}{2}}$.

Let θ, θ_1, θ_2 be the angles made by the tangents to the ellipse, and trajectory with each other and with the axis of x. By trigonometry, we have

$$(1) \qquad \tan\theta=\frac{\tan\theta_1-\tan\theta_2}{1+\tan\theta_1\tan\theta_2},$$

since $\tan\theta=\tan(\theta_1-\theta_2)$.

But $\tan\theta_1=\frac{dy}{dx}=-\frac{ax}{by}$ form the equation to the family of

ellipses, and $\tan\theta_2 = \frac{dy}{dx}$ as referred to the trajectory; and since $\tan\theta = \frac{y}{x}$, (1) becomes, by substitution and reduction,

$$(ax^2 + by^2)\,dx - (a-b)\,xy\,dy = 0;$$

$\therefore\ \frac{dx}{x} - cy\,dy = 0$, or, by integration, $\log x - \frac{cy^2}{2} = \log C$, or, passing to exponentials, $x = Ce^{\frac{cy^2}{2}}$.

This solution was substantially given by all the competitors.

SOLUTION OF NUMERICAL EQUATIONS OF THE THIRD DEGREE.

By PROF. J. C. PORTER, Liberal Institute, Clinton, N. Y.

THE following method of obtaining the approximate roots of cubic equations is analogous to HORNER's, in being based upon the principle of successive transformations. But in obtaining the derived polynomials the common method of synthetical division is departed from, the trial divisors, corrections, and complete divisors being expressed by formulas, which give to the operation a modified form. This modification is supposed to possess the following advantages: —

1. The operation is somewhat abbreviated, even in the absence of decimal contraction.

2. The application of decimal contraction is more perfect, especially in equations having extended decimal coefficients.

3. In the several columns, the terms which correspond to any figure of the root are all situated between the same two horizontal lines, — an arrangement which facilitates a review of the work and the correction of mistakes.

Let us assume, for the general form,

$$(1) \qquad y^3 + sy^2 + py = q;$$

and put $y = m + r$, in which m represents the initial figure or figures of the root, and r the remaining part. Substituting this value of y in equation (1), and factoring with reference to r, we have,

$$(2) \quad (m^3 + s m^2 + p m) + (3 m^2 + 2 s m + p) r + (3 m + s + r) r^2 = q.$$

Let
$$\begin{aligned} \delta &= m^3 + s m^2 + p m, \\ \delta' &= 3 m^2 + 2 s m + p, \\ \delta'' &= 3 m + s + r; \end{aligned}$$

then equation (2) becomes

$$\delta + \delta' r + \delta'' r^2 = q;$$

or,

$$(3) \qquad \delta + (\delta' + \delta'' r) r = q.$$

From this equation we perceive that, if δ be subtracted from the absolute term, q, the remainder may be divided by δ', as a trial divisor, to obtain r, the next figure of the root; the divisor may then be completed by the addition of $\delta'' r$. It is evident, also, that the new trial divisor for obtaining r_1, the next figure of the root, will be what δ' will become when $m + r$ is substituted for m. This we find to be

$$\begin{aligned} (3 m^2 + 2 s m + p) + 2 (3 m + s + r) r + r^2 &= \delta' + 2 \delta'' r + r^2 \\ &= \delta' + \delta'' r + (\delta'' r + r^2). \end{aligned}$$

To correct this trial divisor we must use, instead of δ'', what δ'' becomes when $m + r$ is substituted for m, and r_1 for r; this we find to be

$$3 m + 3 r + s + r_1 = 3 m + r + s + (2 r + r_1) = \delta'' + (2 r + r_1).$$

To recapitulate, we have,

δ = first subtrahend.

δ' = trial divisor.

$$\begin{array}{ll}
\delta'' & = \text{first factor of correction.} \\
\delta'' r & = \text{correction.} \\
\delta' + \delta'' r & = \text{complete divisor.} \\
\delta' + \delta'' r + \delta'' r + r^2 & = \text{new trial divisor.} \\
\delta'' + 2r + r_1 & = \text{first factor of new correction.} \\
(\delta'' + 2r + r_1)\, r_1 & = \text{new correction.}
\end{array}$$

It will be seen by inspecting these formulas, —

1st. That the new trial divisor is formed by adding to the last complete divisor the last correction and the square of the last figure of the root.

2d. The first factor of the new correction is formed by adding to the first factor of the former correction twice the former figure of the root plus the new figure. In all cases *algebraical* addition is understood.

Let it be required to find one value of y in the equation

$$y^3 - 4y^2 + 5y = 12.$$

We find, by trial, the initial figure, $3 = m$; whence we obtain $\delta = 6$, and $q - \delta = 6.000$, the first dividend. We next obtain the trial divisor, $\delta' = 8.00$, and the second figure of the root, $.5 = r$. We then find $\delta'' = 5.5$, $\delta'' r = 2.75$, whence we obtain the complete divisor 10.75, the subtrahend 5.375, and a new dividend, .625000. Next, adding 2.75, the last correction .25, the square of the last figure of the root, and 10.75, the last complete divisor, we have 13.7500, the new trial divisor. Having next obtained 4, the third figure of the root, we annex it to twice the former figure, making 1.04, which we add to the 5.5, and thus obtain 6.54, the first factor of the new correction; and so on, as in the following operation: —

			3.544 +, root.
			12
δ''	$\delta'' r$	δ'	6
5.5	2.75	8.00	6.000
1.04	.25	10.75	5.375
6.54	.2616	13.7500	.625000
84	16	14.0116	.560464
6.624	.026496	14.274800	64536000
		14.301296	57205184
			7330816

In practice, it will be found that the location of the corresponding terms in columns $\delta'' r$ and δ', admits of the requisite facility in addition.

To continue the operation by decimal contraction, we observe that in any application of decimal contraction every multiplicand should have at least one redundant figure; that is, one place more than is necessary to obtain the required number of places in the product. This figure may be multiplied mentally, the tens of the product being carried to the product of the next figure, according to the common rule for contracted decimal multiplication. The suggestions which follow will enable us to secure this condition.

1st. For each new figure in the root, the terms in the operation extend to the right 1 place in the column marked δ'', 2 places in the column marked δ', and 3 places in the column of dividends. Hence,

2d. If at any point we omit to extend the dividend to a new period, we must *shorten* the corresponding term in column δ' 1 place, and the corresponding term in column δ'' 2 places.

3d. If, however, for the first contraction in columns δ' and δ'', we simply omit the *extended parts*, and afterward contract according to the precept just given, each contracted multiplicand will have *one redundant figure.*

It will be observed, that, after the contraction commences, the square of the last figure in the root is not used in forming the new trial divisors, for it is of an order inferior to the lowest in the divisor to be found.

In accordance with these suggestions, the entire operation will appear as follows, decimal points being omitted.

3.5445115283 $\pm$, root.

			12
δ''	$\delta'' r$	δ'	6
55	275	800	6000
104	25	1075	5375
654	2216	137500	625000
84	16	140116	560464
6624	26496	14274800	64536000
85	16	14301296	57205184
6632	3316	14327808	7330816
		14331124	7165562
66	7	1433444	165254
		1433451	143345
		143346	21909
			14335
		14335	7574
			7168
		1434	406
			287
		143	119
			114
		14	5
			4

Let it now be required to find one root of the equation,

$$y^3 + .34785634\,y^2 - 1.03562855\,y = -65.85732648.$$

In examples like this it will be convenient to write the terms in column $\delta'' r$ with units of like order in the same column.

Having found $m = -4$, and computed $\delta = -54.29178436$, and $\delta' = 44.18152073$, the operation will appear as follows: —

			—4.245574430 ±, root.
			—65.85732648
δ''	$\delta'' r$	δ'	—54.29178436
—11.85214366	+2.37042873	+44.18152073	—11.56554212
.44	4	46.55194946	9.31038989
12.292144	.4916858	48.9223782	2.25515223
85	16	49.4140640	1.97656256
12.3771	.061886	49.907350	.27858967
105	25	49.969236	.24984618
12.39	.00620	50.03115	2874349
		50.03735	2501868
12.	.0008	50.0436	372481
		50.0444	350311
		50.045	22170
			20018
		50.05	2152
			2002
		50.1	150
			150
		50.	0

The root thus obtained will always be correct to as many places as are found in any term in the column of dividends.

Geometrical Method.

The formulas for these operations may be deduced geometrically, by a process very similar to that required for the corresponding rule for cube root. For this purpose, let the following problem be proposed: —

Required the dimensions of a rectangular parallelopipedon whose breadth and length exceed its height by d and d' respectively, and whose solidity is g.

Let $y =$ the height; then $y + d =$ the breadth, and $y + d' =$ the length. Multiplying together the three dimensions, we have

(1) $$y^3 + (d + d')y^2 + dd'y = q.$$

Put $s = d + d'$, and $p = dd'$; then

(2) $$y^3 + sy^2 + py = q,$$

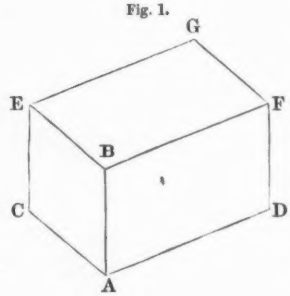
the general form employed in the foregoing pages.

We will suppose the required root of this equation to be irrational, and let $m =$ the initial figure. Since the entire root is to be the height of a right prism whose several dimensions are as given in the problem, we will form a right prism (Fig. 1), having $AB = m$, the height; $AC = m + d$, the breadth; and $AD = m + d'$, the length. The solidity of the prism is

$$m(m+d)(m+d') = m^3 + (d+d')m^2 + dd'm = m^3 + sm^2 + pm = \delta.$$

Subtracting the solid contents of the prism from that of the required prism, we have, for a remainder, $q - \delta$. If we now enlarge our prism (Fig. 1) by the addition of this remainder, its solidity will be that of the required prism. To preserve the given relations, the addition must be made upon three adjacent sides or faces, in the form of a covering of uniform thickness. The addition will therefore be composed of three flat prisms to cover the three faces (Fig. 2), and three oblong prisms and one cube to fill the vacancies at the edges and corner (Fig. 3). Now the thickness of this enlargement will be additional height of the prism, and consequently the remaining part of the root. Hence, to obtain a second figure of the root, we may divide the remainder, $q - \delta$, by the sum of the areas of the three flat prisms (Fig. 2), as a *trial divi-*

Fig. 1.



sor. These areas are the same as the three faces of Fig. 1, and are as follows: —

$$ABEC = m(m+d) \qquad = m^2 + dm.$$
$$ABFD = m(m+d') \qquad = m^2 + d'm,$$
$$BFGE = (m+d)(m+d') = m^2 + (d+d')m + dd'.$$

Adding these areas, we find the sum to be

$$3m^2 + 2sm + p = \delta'.$$

Represent by r the figure obtained by this trial divisor, which will express the common width of the four remaining prisms (Fig. 3).

Fig. 2.

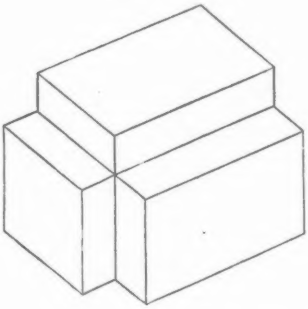
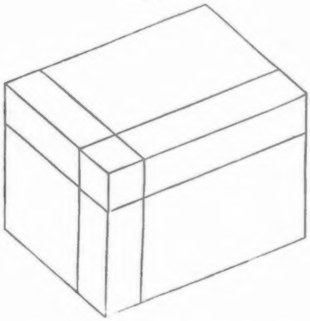


Fig. 3.



To complete the divisor, we must add to the trial divisor the sum of the areas of these four remaining prisms. The aggregate length of these prisms is $m + (m+d) + (m+d') + r = 3m + s + r = \delta''$; their area will therefore be $\delta'' r$, which is the correction required.

Having completed the divisor, and formed a new remainder, we must proceed to make a new enlargement of the prism. The trial divisor to obtain the thickness of this second enlargement, or the next figure of the root, will be the area of three new flat prisms to cover the three faces of the prism already formed; and this surface (Fig. 3), is composed of one face of each of the former flat prisms, two faces of each of the oblong prisms, and three faces of

the cube at the corner. But we have in the last complete divisor one face of each of the flat prisms, oblong prisms, and cube; and in the last correction, one face of each of the oblong prisms and cube; and in the square of the last figure of the root, a third face of the cube; and these are the three quantities which were added together to form the new trial divisor, as before explained.

We have thus derived the algebraical formulas before used, by a process entirely geometrical. To adapt this explanation of the rule to the case of a negative root, we have only to give the prism the proper location relative to three rectangular co-ordinates, according to the principles of analytical geometry.

MOTION ON AN INCLINED PLANE.

(*Prepared for Elementary Instruction.*)

By Thomas Sherwin, Principal, English High School, Boston.

We have seen, in the Theory of the Inclined Plane, No. V. Vol. II., that, when the power acts parallel to the plane, θ being the angle of the elevation, h, the height, and l, the length of the plane, R being 1, $P : W = \sin\theta : 1 = h : l$.

Now P, in this case, is equal to that part of the weight which urges the body down the plane. If, then, the body were unrestrained, it would, the elevation of the plane remaining the same, be subject to a constant force which would produce a uniformly accelerated velocity; and it is evident that this accelerating force bears to that of a body falling freely the same ratio that the equilibrating power bears to the weight. Therefore, calling g' the accelerating force down the plane, we have $g' : g = \sin\theta : 1 = h : l$; $\therefore$

(1) $$g' = g\sin\theta = \frac{hg}{l}.$$

Since the force which urges a body down a plane is the same as that which causes a body to fall freely, except that it is of less intensity, it follows that the motion of a body down such a plane is subject to the same laws as that of falling bodies. Hence, on the same plane,

(2) $$t : t' = v : v',$$

(3) $$s : s' = t^2 : t'^2 = v^2 : v'^2,$$

(4) $$s_1 : s_2 : s_3, \&c. = 1 : 3 : 5, \&c. ;$$

that is,

The times are as the final velocities ; the spaces are as the squares of the times, or as the squares of the final velocities ; and the spaces described in equal successive portions of time are as the successive odd numbers, 1, 3, 5, 7, &c.

Since $g' = g \sin\theta = \frac{hg}{l}$, if we substitute these values of g' instead of g in the formulæ for falling bodies, we shall have corresponding formulæ for the motion of bodies on inclined planes. Hence,

(5) $$v = gt \sin\theta = \frac{hgt}{l};$$ (6) $$t = \frac{v}{g\sin\theta} = \frac{lv}{hg}$$

(7) $$s = \tfrac{1}{2} g t^2 \sin\theta = \tfrac{1}{2}\frac{hgt^2}{l};$$ (8) $$t = \sqrt{\frac{2s}{g\sin\theta}} = \sqrt{\frac{2ls}{hg}};$$

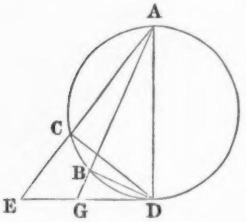
(9) $$s = \frac{v^2}{2g\sin\theta} = \frac{lv^2}{2hg};$$ (10) $$v = \sqrt{2gs\sin\theta} = \sqrt{\frac{2hgs}{l}}.$$

Formulæ may be easily obtained for bodies *projected* down or up inclined planes. By reference to Articles IV. and V. of Falling Bodies, the student will find no difficulty in obtaining these formulæ.

II. Let u be the velocity which a body acquires in falling freely through the height h, and v the velocity which the body acquires in moving down l, the length of a plane. Then $u = \sqrt{2gh}$, and, from (10), since s equals l, $v = \sqrt{\frac{2hgl}{l}} = \sqrt{2gh}$; $\therefore u = v$. Hence,

A body acquires the same velocity in moving down the length of an inclined plane, as it would acquire in falling freely through the height of the plane.

Let t and t' be the times in which a body would descend the inclined planes AG and AE as far as to the perpendiculars BD and CD drawn from D to these planes, and let the elevation of these planes be θ and θ'. Then, since AB and AC are the spaces, we have



$$AB = \tfrac{1}{2} g t^2 \sin\theta, \quad \text{and} \quad AC = \tfrac{1}{2} g t'^2 \sin\theta'.$$

But, in the right-angled triangles ABD and ACD,

$$AB = AD \cos BAD = AD \sin\theta,$$

and $$AC = AD \cos CAD = AD \sin\theta';$$

$$\therefore\ AD \sin\theta = \tfrac{1}{2} g t^2 \sin\theta, \qquad \therefore\ (1)\ AD = \tfrac{1}{2} g t^2;$$

and $$AD \sin\theta' = \tfrac{1}{2} g t'^2 \sin\theta', \qquad \therefore\ (2)\ AD = \tfrac{1}{2} g t'^2;$$

hence, from (1) and (2),

$$\tfrac{1}{2} g t^2 = \tfrac{1}{2} g t'^2, \qquad \therefore\ t = t',$$

Hence, the times of descent through AB and AC are the same.

If upon AD as a diameter the circumference of a circle be described, it will pass through the vertices of the right angles B and C, and AB and AC are chords drawn from A. Hence:

If any number of chords are drawn from either extremity of the vertical diameter of a circle, a heavy body would move down these chords or fall freely through the vertical diameter in the same time.

III. If we call l and l' the lengths of two inclined planes, h and h' their heights, and t and t' the times of descent through these lengths since l and l' are the spaces, we have from formula (8), and from the analogous formula $t' = \sqrt{\frac{2\,l'\,s}{h'\,g}}$, by putting l and l' for s,

$$t = \frac{l\sqrt{2}}{\sqrt{h\,g}}, \qquad \text{and} \qquad t' = \frac{l'\sqrt{2}}{\sqrt{h'\,g}}; \qquad \therefore\ t : t' = \frac{l\sqrt{2}}{\sqrt{g\,h}} : \frac{l'\sqrt{2}}{\sqrt{g\,h'}}.$$

Dividing both terms of the last ratio by $\sqrt{2}$, and multiplying by $\sqrt{g}$, we have

(1) $$t : t' = \frac{l}{\sqrt{h}} : \frac{l'}{\sqrt{h'}}.$$

Hence,

In two different inclined planes, the times of descent through the lengths of the planes are as the lengths divided by the square roots of the heights.

If $h = h'$, proportion (1) becomes

(2) $$t : t' = l : l'.$$

If in the same proportion $l = l'$, it becomes $t : t' = \frac{1}{\sqrt{h}} : \frac{1}{\sqrt{h'}}$;

or, multiplying both terms of the last ratio by $\sqrt{h}$ and $\sqrt{h'}$,

(3) $$t : t' = \sqrt{h'} : \sqrt{h}.$$

Hence, from (2) and (3),

When in two inclined planes the heights are the same, the times of descent are as the lengths; and when the lengths are the same, the times of descent are inversely as the square root of the heights.

It may be proved in a similar way that the times of descent down the chords CD and BD are equal, and as the diameter itself may be regarded as a chord, the times of descent would each be equal to that in which a body would fall freely through this diameter.

THE THEORY OF THE GYROSCOPE.

By Theodore G. Ellis, Civil Engineer, Boston.

The interest excited in this little instrument, by its wonderful power of seeming to overcome the force of gravity, has induced the following attempt at an explanation of its principles in as concise and simple a manner as possible, with a hope that it may call

out the investigations of others who have made experiments or researches upon this interesting subject.

To have an apparatus in every way suited to our investigations, with a view to their being more readily understood, let us suppose the ordinary Gyroscope, consisting of a wheel, axle, and supporting ring, to be hung in another ring by pivots at right angles to the direction of the axis of the wheel; and this second ring to be supported also by pivots, at right angles to the former ones, upon a stand capable of turning upon a vertical axis, which, if produced, would pass through the centre of the wheel. The lines joining the opposite points in the rings would also pass through the centre of the wheel.

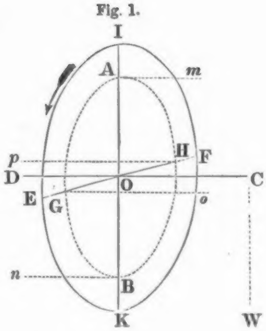
We then have the Gyroscope hung in gimbals precisely like a mariner's compass, except the additional contrivance for turning upon a vertical axis.

This mode of hanging the wheel admits of its motion freely in all possible directions around its centre, except when the horizontal axis of the wheel, the vertical axis, and all the points of suspension lie in the same vertical plane. In this position, motion of the wheel round its centre in that plane cannot take place. In our present examination we will suppose this position not to occur, as in all others motion will be free in all directions around the centre of the wheel.

For the sake of simplicity, we will also suppose the revolving wheel to be a circular disk, suspended on an axis of no appreciable weight compared with the weight of the wheel.

Let $IEKF$ (Fig. 1) be a circular disk, suspended as above described, and revolving with a given velocity in the direction of the arrow. Let DC and EF be horizontal, and IK vertical. At the end, C, of the axis, DC, let there be a force, W, acting downwards parallel to IK.

Fig. 1.



Then will each particle of matter in the wheel above the horizontal diameter, *E F*, have a tendency to move in the direction *A m* parallel to *D C*, and each particle of matter below *E F* will have a tendency to move in the opposite direction, *B n*.

We see, therefore, that at any one instant of time, all the particles of matter above *C F* will be acted upon by a force in the direction *A m*, but when the wheel has performed a half revolution, these same particles are acted upon by a force in the direction *B n*, exactly opposing and neutralizing the former. And so with the half circle below *E F*, it is at one instant acted upon in the direction *B n*, and the next, when it has made half a revolution, it is acted upon in the direction *A m*. Thus each particle of which the wheel is composed receives in rapid succession impulses in opposite directions, which, owing to its inertia and the rigidity of the wheel, it constantly resists.

Let us take a particle in the vertical diameter, as at *A*, and follow it round the circle. During its passage through the quadrant *A G* it is continually acted upon by a succession of forces tending to move it in the direction *A m*, parallel to *D C*. After passing the horizontal diameter *E F*, from *G* to *B* it is acted upon by a succession of forces exactly equal to those above the horizontal diameter, but acting in an opposite direction, *B n*, so that any motion or momentum acquired in passing through *A G* will be exactly counterbalanced and wholly destroyed in passing through *G B*, so far as relates to the directions *A m* and *B n*; so that if the particle was at rest at *A*, relative to those directions, it will be again at rest on arriving at *B*.

The same effects as above described also evidently take place while the particle passes through *B H A*.

What is true for any one particle must be true for every other, and consequently for the whole wheel, and the forces above and below *E F* just balancing each other, no motion around *E F* results from the force, *W*, while the disk is revolving with a uniform velocity.

Having thus shown that the force, *W*, will not produce motion about the horizontal diameter, *E F*, which is at right angles to *D C*, the axis, *D C*, will remain in a horizontal plane. Motion, therefore, of the plane, *I E K F*, can only occur about the vertical axis *I K*, as we have the point *O* fixed by construction, being the centre of all motion of which the wheel is capable.

Let us now examine the effect of the forces acting on any particle of matter passing around the circle, with regard to the axis *I K*.

From *A* to *G* we have seen that it is acted upon by a force tending to move it in the direction *A m*; and from *G* to *B* by a force tending to move it in the direction *B n*; and that these forces are equal and act in opposite directions. Now if a body is acted upon by any force for a definite period of time, motion will be produced in the direction of the force, and a certain velocity will be acquired; if an equal force then act on the same body in an opposite direction, for the same period of time, the velocity acquired is gradually retarded and the body left at rest.

This is precisely the case with a particle of matter passing through *A G B* or through *B H A*. A force tends to accelerate its motion in the direction *A m* from *A* to *G*, and from *G* to *B* it is gradually retarded by a force acting in the direction *B n*. So also through *B H A* a force tends to accelerate its motion in the direction *B n*, while passing through *B H*, and through *H A* it is again retarded and brought to rest by the force acting in the direction *A m*.

As the forces producing these motions around *I K* are equal on opposite sides of *I K*, and their centres of moment are equally

distant, they will produce motion round IK as an axis independently of the point, O, being fixed by construction.

Let us for a moment consider what would be the effect if this circular motion around IK could not take place. Suppose, for instance, that the axis CD were confined to a vertical plane, and to be in a horizontal position. In this case no horizontal circular motion could occur, so that all particles of matter of which the wheel is composed passing through EO downwards and through OF upwards would be brought to relative rest with regard to a horizontal plane. But immediately on passing EO and OF they would be acted upon by the opposite forces in the directions Am and Bn; motion would immediately take place around EF, and the weight W would fall.

We thus see that this motion around IK not only takes place, but is absolutely necessary to the equilibrium maintained at A and B.

We will now proceed to ascertain the time of rotation around IK. Suppose the circular disk to be divided into an indefinite number of small sectors, then will the centre of gravity of each be distant from the centre of the disk by two thirds of the radius of the disk. For convenience of demonstration we will consider all the weight of the disk to be in the circle joining these centres of gravity, calling the radius of this circle "radius of gravity," and limiting ourselves to such reasoning as will be consistent with the above supposition.

This method of considering the weight of the wheel to be in its circle of gravity has also the advantage of being applicable to wheels of all forms.

As the forces acting in the quadrants GB and HA are solely employed in bringing to a state of rest the motion acquired by each particle of matter in the quadrants AG and BH, they exercise no

influence upon the rotation of the wheel around IK, which rotation is therefore solely due to the motion acquired in the quadrants AG and BH.

Again, each particle of matter of which the wheel is composed is limited to the plane of revolution of the wheel, so that in passing through AG or BH it will not admit of being uniformly accelerated, but will move in successive instants of time through spaces proportional to the sine of the angular distance from IK. But as in passing through AG or BH each quantity of motion lost by one particle on account of its being confined to the plane of the wheel, is distributed among all others lying on the same side of IK, the total amount of motion for any one particle is the same as though it were uniformly accelerated while passing through AG or BH.

Referring again to Fig. 1, let

$r = AO =$ radius of gravity.
$a = OC$.
$t =$ time employed in revolving.
$g =$ force of gravity $= 32.2$.
$\pi =$ ratio of circumference to diameter.
$W =$ force, in units of weight, at C.
$W' =$ weight of wheel.

To find the accelerating force, F, acting on each particle of the wheel and turning it round IK, we proceed as follows:—

We have $W =$ number of units of weight acting at a radius of a tending to turn the wheel on GH. We have also the distance from O to the centres of moment of the semicircles

$$GAH \text{ and } GBH = \frac{2r}{\pi}.$$

Denoting by F the force in units of weight acting on the centres of moment of the semicircles GAH, GBH, we have

$$F \times \frac{2\,r}{\pi} = a\,W, \qquad\qquad F = \frac{\pi\,a\,W}{2\,r}.$$

If then the weights W and W' were equal, we should have the accelerating force, F, acting on the wheel and the force of gravity, g, proportional to the pressures F and W, or

$$F : g = \frac{\pi\,a\,W}{2\,r} : W, \qquad\qquad F = \frac{\pi\,g\,a}{2\,r}.$$

But if the weights W and W' are different, the force F will vary inversely as the proportion of W' to W, or directly as $\frac{W}{W'}$; hence,

$$F = \frac{\pi\,g\,a}{2\,r} \times \frac{W}{W'}.$$

By the law of bodies moving under the influence of a continuous force, the space described equals $\frac{1}{2}\,t^2\,F$. We have, therefore,

(1) $$s = \tfrac{1}{2}\,t^2 \times \frac{\pi\,g\,a\,W}{2\,r\,W'} = \frac{\pi\,g\,t^2\,a\,W}{4\,r\,W'}.$$

Representing by T the time required for one whole rotation about IK, we have,

$$T : t = 2\,\pi\,r : \frac{\pi\,g\,t^2\,a\,W}{4\,r\,W'},$$

(2) $$T = \frac{8\,r^2\,W'}{g\,t\,a\,W}.$$

If $n =$ number of revolutions of wheel per second, then $4\,t$, equal to the time of one revolution, $= \frac{1}{n}$, and $t = \frac{1}{4\,n}$. Substituting in above formula, we have

(3) $$T = \frac{32\,n\,r^2\,W'}{g\,a\,W}.$$

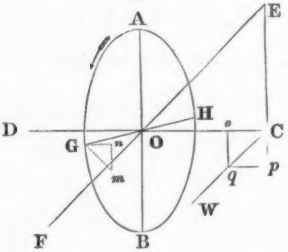
In these formulæ the units of length and time are feet and seconds. Of weight, any unit may be taken.

Let us now consider the case where the axis CD is not horizontal, but inclined at any angle to the horizon except vertical, when the force W would pass through the centre O.

We will suppose the Gyroscope suspended as before, and revolving with a uniform velocity.

Let $A\,B$, $C\,D$, and $G\,H$ (Fig. 2) represent the same lines as before, $A\,B$ and $C\,D$ lying in the same vertical plane, and $G\,H$ being horizontal. Let W be the weight hanging vertically from the end of the inclined axis $C\,D$. Draw the vertical line $E\,F$ through the centre of the wheel, O. Represent the weight W by $C\,q$, and resolve this force into two others lying in the vertical plane which passes through $E\,F$ and $C\,W$; one force being at right angles to, and the other coinciding with the axis $C\,D$. Completing the parallelogram we have one force $= C\,p$, and the other $= C\,o$. Produce $C\,p$ to E.

Fig. 2.



The force $C\,o$ acting through the centre of the wheel has no tendency to turn it in any direction. The force $C\,p$, acting at right angles to $C\,D$, and tending to turn the wheel on the horizontal line $G\,H$, at right angles to $C\,D$, would, as we have before shown, cause the wheel to rotate on $A\,B$ if it continued to act in a direction parallel to $C\,p$. That is, the point G would move in the direction $G\,n$. But this force is constantly changing its direction, as it always lies in a vertical plane passing through $E\,F$; and as it constantly tends to turn the wheel in the vertical plane in which it lies, the line $G\,H$, being always at right angles to this plane, must remain horizontal, and the points G and H will move in a horizontal circle around O. The angle $E\,O\,C$ will remain constant, and the wheel will rotate about the vertical line $E\,F$ parallel to $C\,W$.

Let us now suppose the point C to revolve around $E\,F$ the indefinitely small distance $C\,x$. In passing through this indefinitely small space we may consider two forces to have been acting at the point

C, one of which is continually parallel to Cp, causing the wheel to rotate on AB, and the other operating to turn AB and GH on CD as an axis through the small angle AEx.

From G lay off Gn parallel to DC equal to the distance the point G would move under the influence of the parallel force Cp. Draw nm perpendicular to Gn, and make Gm horizontal. Then will Gn and nm represent the distances the point G would be moved by the two forces acting at the point C. Gm is the resultant, and represents the distance actually passed through by the point G.

In the triangles Gnm and Cqp we have the sides of one respectively perpendicular to the sides of the other in direction; therefore they are similar, and we have

$$Gn : Gm = Cp : Cq.$$

We see, from equation (1) previously obtained, that the space described in rotating by the influence of a force acting at right angles to CD is proportional to the weight W. If, therefore, Gn is the distance revolved under the influence of the force Cp, Gm would be the distance revolved under the influence of the force $Cq = W$ acting at right angles to CD, and is therefore the same distance that would be described by the point G when CD is horizontal.

As the proof of the above would be similar for any inclination below the horizontal line, we see that for all inclinations of CD we have the angular velocity and time of rotation about the vertical axis the same.

From the foregoing we may deduce the general law of the Gyroscope, which will account for all the phenomena exhibited by different experiments, viz.: —

A revolving body, acted upon by a force passing through its axis of revolution, will rotate with a new motion round an axis passing through its centre of motion parallel to the direction of the force.

The time of rotation will be directly proportional to the velocity of revolution, the square of the radius of gravity in the plane of revolution, and the weight of the body; and inversely proportional to the force, and the distance on the axis from the centre of the body to the line in which the force acts.

The case in which resistance occurs to the rotating motion comes under the above general law by considering the reaction from the resistance to be a force.

Let us see how this law operates in the ordinary form of toy gyroscope, apparently supporting its own weight and rotating around a fixed support.

Here we have the whole weight of the wheel and ring acting downwards by the force of gravity; but as the wheel does not yield to its influence, on account of its being obliged to turn out of its revolving plane to fall, which motion we have seen is resisted by a revolving body, the whole weight is transferred to the point of support, which reacts in an upward direction with an equal force to sustain it. Here we have a force acting on the end of the axle, equal to the weight of the wheel and ring, tending to turn the revolving wheel on its horizontal diameter. This we have seen will cause it to rotate on a vertical axis. But as the point of suspension is fixed, the rotation of the wheel on a vertical axis reacts on the point of support, and the wheel itself moves instead of the point at which the upward force acts. And as one rotation of the wheel on its vertical axis will cause one complete turn around the point of support, the time employed in making the latter will be given by the formula (3) previously found, viz.: —

$$T = \frac{32\, n\, r^2\, W'}{g\, a\, W}.$$

In this case W is the weight of the wheel and ring, and W' the weight of the wheel alone.

When the Gyroscope is suspended by a cord instead of the fixed support, it also appears to turn about the point of suspension. The reason of this is, that the whole weight of the apparatus being transferred as before to the point of suspension, that point is in stable equilibrium only when the cord hangs vertically, and the resistance to motion, to any distance out of this vertical line, is very much greater than the resistance of the air to the passage of the revolving wheel, so that the slight reaction on the point of suspension necessary to cause rotation round this point will move it but a small distance, and cause the cord to vary but slightly from a vertical position.

Finally, let us examine into the cause of the gradual falling of the weight attached to the axis of the Gyroscope.

We have already seen that when the wheel revolves uniformly and without friction, there is no yielding in the direction of the force, it being entirely expended in producing the rotation round the vertical axis.

That the falling of the weight is due to friction, including under this term all causes of resistance, may be shown as follows.

Let us consider the friction as operating in two ways: First, by causing a retardation of the velocity of the revolving wheel, and, secondly, by the resistance it offers to motion at the point of support.

It is evident, from our foregoing reasoning, that if a particle of matter is longer in passing through the second quadrant on either side of AB than through the first, it will more than have its motion arrested, and will acquire a motion in an opposite direction, and will not be at rest on arriving at A or B; and by just so much distance as it has moved through will the wheel be turned on its horizontal diameter, thus allowing the weight to fall through a corresponding angle.

We have also seen that any force arresting the rotation about AB causes the weight to fall. The friction upon the point of support, though not actually stopping the rotation of the wheel, operates in the same manner. The particles of matter in the first quadrant are prevented from acquiring the full amount of motion due to the force, and thus in the second quadrant the motion is more than counterbalanced, so that instead of being at rest at the points A and B, motion is acquired in the opposite direction; and by just the distance moved through, after coming to rest before arriving at A or B, will the wheel be turned on its horizontal diameter and the weight will fall by a corresponding amount.

A COMPLETE CATALOGUE OF THE WRITINGS OF SIR JOHN HERSCHEL.

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1812. NICHOLSON'S Journal, XXXI. p. 133, No. 142. Analytical Formulæ by "A Lover of the Modern Analysis."

1812. Ibid., XXXII. p. 13. A letter signed "Analyticus." (Trigonometrical Formulæ.)

1812. LEYBOURNE'S Mathematical Repository for 1814, Part II p 58. New Properties of the Conic Sections.

1812. Phil. Trans. Royal Soc. London, 1813. On a Remarkable Application of COTES'S Theorem.

1813. On Trigonometrical Series, ... &c., ... Mem. Analyt. Soc. Cambridge.

1813. On Equations of Differences and their Application to the Determination of Functions under Given Conditions. Mem. Analyt. Soc. Cambridge. N. B. — The latter portion of this Part III. on Functional Equations is all wrong.

1814. Phil. Trans. Royal Soc. London, 1814. Consideration of Various Points of Analysis.

1814. Problems 362, 363, 364, 365, 366, 367, 368, 369, LEYBOURNE'S Mathematical Repository. — Also (I believe somewhere in that Repository), Properties of the Catenary, analogous to the Trigonometrical Formulæ in the Circle, and a number of other problems, &c., in other parts of that Repository.

1815. Phil. Trans. Royal Soc. London, 1816. On the Development of Exponential Functions together with several new Theorems relating to Finite Differences.

1816. The notes N, O, P, Q, in the Translation of LACROIX'S 8vo Treatise on the Differential and Integral Calculus, by Messrs. PEACOCK, HERSCHEL, and BABBAGE, with an original Appendix on Differences.

1817. The article "Isoperimetrical Problems" in the Edinburgh Cyclopædia.

1818. The article "Mathematics in the Edinburgh Cyclopædia, XIII., Parts 1, 2.

1818. On Circulating Functions, and on the Integration of a Class of Equations of Finite Differences into which they enter as Coefficients. Read before the Royal Soc., Feb. 19. Phil. Trans.

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1819. On the Hyposulphurous Acid and its Compounds. BREWSTER'S Edinburgh Phil. Journal, Vol I., p. 8.

1819. A Notice in BREWSTER and JAMIESON'S Journal of Sir W. HERSCHEL'S Paper on the Distance of the Fixed Stars.

1819. On some Optical Phenomena exhibited by Mother of Pearl, dependent on its Internal Structure. BREWSTER and JAMIESON's Journal, I. p. 396.
1819. On the Application of a new Mode of Analysis to the Theory and Summation of certain extensive Classes of Series. BREWSTER and JAMIESON's Edinburgh Phil. Journal, Vol. II p. 23.
1819. Some Additional Facts relating to the Habitudes of the Hyposulphurous Acid and its Union with Metallic Oxides. BREWSTER and JAMIESON's Phil. Journal, Vol. II. p. 154.
1819. Refractive Indices of Hyposulphite of Soda and Silver and of Nitrate of Lead. BREWSTER and JAMIESON's Phil. Journal, Vol. II. p. 184.
1820. A Collection of Examples of Applications of the Calculus of Finite Differences.
1820. On certain Remarkable Instances of Deviation from Newton's Scale in the Tints developed by Crystals with one Axis of Double Refraction on Exposure to Polarized Light. Cambridge University Phil. Trans., Vol. I.
1820. On the Rotation Impressed by Plates of Rock Crystal on the Planes of Polarization of the Rays of Light as connected with Certain Peculiarities in its Crystallization. Cambridge University Phil. Trans , Vol. I.
1820. On the Reduction of Certain Classes of Functional Equations to Equations of Finite Differences. Read March 6th. Trans. of the Cambridge Phil. Soc.
1821. On a Remarkable Peculiarity in the Law of Extraordinary Refraction of differently colored Rays exhibited by certain varieties of Apophyllite. Cambridge University Phil. Trans., Vol. I.
1821. On the Aberrations of Compound Lenses and Object Glasses. Phil. Trans. Royal Soc. London.
1821. Auszug Einer Briefe von Sir J. F. W. H. Astron. Nachr., No. 2, p. 30.
1821. On the Separation of Iron from other Metals. Read April 5th, Phil. Trans.
1821. A Letter to Dr. BREWSTER on the Practical Rules for the Determination of the Radii of a Double Achromatic Object Glass. Edinburgh Phil. Journal.
1822. On the Absorption of Light by Colored Media, and on the Colors of the Prismatic Spectrum exhibited by various Flames, with an Account of a ready Mode of obtaining the absolute Dispersive Power of any Medium by direct Experiment. Edinburgh Phil. Trans., Vol. IX. p. 446.
1823. Auszug Einer Briefe, &c. Astron. Nachr., No. 44.
1823. The Article "Physical Astronomy" in the Encyclopædia Metropolitana.
1823. Subsidiary Tables for facilitating the Computations of Annual Tables of the Apparent Places of 46 Principal Fixed Stars computed by order of the Council of the Astron. Soc., &c., &c. Mem. Astron. Soc., Vol. I., published 1824.
1824. Observations of the Apparent Distances and Positions of 380 Double and Triple Stars, by J. F. W. H. and J. SOUTH. Phil. Trans , 1825, Part III.
1824. A Notice of AMICI's Double-Image Micrometer. Notice read to Astron. Soc. Phil. Magazine.
1824. On a New Method of computing Occultations of the Fixed Stars. Communicated in a Letter to CHARLES BABBAGE. Read Jan. 10th, 1823. Mem. Astron. Soc.
1824. On Certain Motions produced in Fluid Conductors when transmitting the Electric Current. (Bakerian Lecture.) Read before the Royal Soc., Feb. 12th. Phil. Trans.
1825. On the Mechanical Effects produced when a Conducting Liquid is electrified in contact with Mercury. BREWSTER's Edinburgh Journal of Science, Vol. II. p. 193.
1825. Analysis of Dr. BREWSTER's Paper on the Monochromatic Lamp, and of the Absorption of the Prismatic Rays by Colored Media, &c. BREWSTER's Edinburgh Journal, Vol. II. p. 344.
1825. Extract of a Letter from J. F. W. HERSCHEL to Dr. RITCHIE. (This contains the first published account of my Principle of Actinometry.) Phil. Jour., Vol. III. p. 107.
1825. The notes signed (H) in Mr. SOUTH's Paper on Double Stars. Phil. Trans. Royal Soc. 1823.
1825. Abstract of a Memoir by M. PLANA. Proceedings of Astron. Soc. London, 1825, Phil. Magazine.
1825. An Account of the Repetition of M. ARAGO's Experiments on the Magnetism manifested by various Substances during the Act of Rotation. Read before the Royal Soc., June 16th. Phil. Trans.
1825. Extract of a Letter to M. SCHUMACHER, dated Aug. 15th, on Mr. FRAUNHOFER's Memoir on the Inferiority of Reflecting Telescopes when compared with Refractors.
1825. Approximate places of 321 new Double and Triple Stars, for January 1st, 1825, with their estimated Angles of Position, Distances, Magnitudes, and other Particulars. Astron. Soc. of London, Vol. II.
1826. An Account of a Series of Observations made in the Summer of the Year 1825, for the Purpose of Determining the Difference of the Meridians of the Royal Observatories of Greenwich and Paris. Read before the Royal Society, Jan. 12th. Phil. Trans.
1826. On the Parallax of the Fixed Stars. Read before the Royal Society, March 9 and 16. Phil. Trans.

1827. Correction of an Error in a Paper on the Parallax of the Fixed Stars. Phil. Trans. Royal Soc.

1827. A Paper read at a Meeting of the Board of Longitude. Printed by order of the Board. On Reform of the Nautical Almanac.

1827. The article "Light" in the Encyclopædia Metropolitana. This has been translated into French and German.

1827. An Address delivered at a Special General Meeting of the Astronomical Society of London, on April 11th, 1827, on presenting the Honorary Medals to F. BAILY, Esq., Lieut. W. S. STRATFORD, R. N., and Col. MARK BEAUFOY.

1827. An Account of Observations made with a Twenty-feet Reflecting Telescope, containing a Second Catalogue of 295 New Double and Triple Stars (reduced to the beginning of 1830); together with some observations of Double Stars previously known. Memoirs of the Astron. Soc. London., Vol. III.

1828. Third series of Observations with a Twenty-feet Reflector, containing a Catalogue of 384 New Double Stars (reduced to the beginning of 1828); together with some Observations of Double Stars previously known. Memoirs of the Astron. Soc. London, Vol. III.

1828. An Address on the Occasion of the Delivery of the Honorary Medals of the Astronomical Society of London, on Feb. 8th, 1828, to Lieut. Gen. Sir T. MACDOUGAL BRISBANE, K.C.B., and JAMES DUNLOP, Esq.

1829. An Address delivered at the Anniversary Meeting of the Astronomical Society of London, Feb. 13th, 1829, on Presenting the Honorary Medals to Rev. W. PEARSON, Professor BESSEL, and Professor SCHUMACHER.

1829. The Article "Sound" in the Encyclopædia Metropolitana, Vol. IV., for 1829-30.

1830. Extract of a Letter from Sir J. F. W. H. to M. QUETELET, on the "Roccellic Acid." Read at the Royal Academy of Sciences, Brussels, May 22d, 1830. Bulletins of the Academy.

1830. On the Study of Natural Philosophy, a Preliminary Discourse to LARDNER'S Cabinet Cyclopædia.

1830. Fourth Series of Observations with a Twenty-feet Reflector; containing the Places, Descriptions, and Measures of 1236 Double Stars (reduced to the beginning of 1830); the greater part of them not previously described. Memoirs of the Astron. Soc. London, Vol. IV.

1832. Fifth Catalogue of Double Stars observed at Slough in the years 1830 and 1831 with the Twenty-feet Reflector; containing the Places, Descriptions, and measured Angles of Positions of 2007 of those Objects, of which 1304 have not been found described in any previous collection; the whole reduced to the Epoch 1830. Communicated to the Royal Astron. Soc., June 7th, 1832. Royal Astron. Soc., Vol. VI.

1832. Micrometrical Measures of 364 Double Stars, with a Seven-feet Equatorial Achromatic Telescope; taken at Slough, in the years 1828, 1829, 1830. Memoirs of the Royal Astron. Soc., Vol V. Part I.

1832. On the Astronomical Causes which may influence Geological Phenomena. Read Dec. 15th, 1830. Trans. Geological Soc. London, 1832.

1832. Observations of BIELA'S Comet. Read Nov. 9, 1832. Royal Astron. Soc., Vol. VI.

1832. On the Action of Light in determining the Precipitation of Muriate of Platina by Lime-water, in a Letter to Dr. DAUBENY. Read before the British Association at Oxford, June 22d. London, Edinburgh, and Dublin Phil. Magazine, New Series, No. 1.

1832. Notice of BIELA'S Comet in a Letter to the "Times," Sept. 26th, 1832.

1832. On BIELA'S Comet, a 2d Letter to the "Times," Sept. 27th, 1832.

1832. Observations of BIELA'S Comet. Memoirs Astron. Soc. London, Vol VI. p. 99.

1832. Observations of δ Cygni, μ Aquarii, α Capricorni, and BIELA'S Comet. Astron. Nachr., No. 236.

1832. Description of a Machine for resolving by Inspection certain important Forms of Transcendental Equations. Read May 7th, 1832. Trans. of the Cambridge Phil. Soc.

1832. On the Investigation of the Orbits of Revolving Double Stars: being a Supplement to a Paper entitled "Micrometrical Measures of 364 Double Stars." Read Jan. 13th, 1832. Royal Astron. Soc., Vol. V.

1832. Lettre adressée à MM., les RÉDACTEURS des Annales de Chimie et de Physique, sur la Séparation de l'Oxide de Fer, et sur un nouveau Procédé pour effectuer la Purification complète de l'Oxide d'Urane; 24 Avril, 1832. Extrait des Annales de Chimie, Mars, 1832.

1833. An Address to the Subscribers to the Windsor and Eton Public Library and Reading-room. Published by KNIGHT, London.

1833. "Notice sur la Manière d'agir de l'Acide Nitrique sur la Fer." Annales de Chimie et de Physique, Tom. IV. p. 89. Lettre à MM., les RÉDACTEURS.

1833. Review of Mrs. SOMERVILLE'S Mechanism of the Heavens, and of NATHANIEL BOWDITCH'S Translation of the "Mécanique Céleste of Laplace" Quarterly Review, No. XCIV.

1833. A Treatise on Astronomy. Cabinet Encyclopædia, Part 43.

1833. Notices of the Elliptic Orbits of ξ Boötis and η Coronæ, with a second Approximation to the Orbit of γ Virginis. Read June 14th, 1833. Memoirs of the Royal Astron. Soc., Vol. VI.

1833. A Letter addressed to R. PHILLIPS, Esq. &c., &c., dated January 12th, 1833, on a Remarkable

Deposition of Ice round the Decaying Stems of Vegetables during Frost. London and Edinburgh Phil. Magazine and Journal.

1833. On the Absorption of Light by Colored Media, viewed in connection with the Undulatory Theory. From the London and Edinburgh Phil. Magazine and Journal of Science.

1833. Observations of Nebulæ and Clusters of Stars, made at Slough, with a Twenty-feet Reflector, between the Years 1825 and 1833. Read Nov. 21, 1833. Phil. Trans.

1834. On the Satellites of Uranus. Read March 14th, 1834. Royal Astron. Soc., Vol. VIII.

1834. A List of Test Objects, principally Double Stars, arranged in Classes, for the Trial of Telescopes in various respects, as to Light, Distinctness, &c. Memoirs of the Royal Astron. Soc., Vol. VIII.

1834. A Second Series of Micrometrical Measures of Double Stars, chiefly performed with the Seven-feet Equatorial at Slough, in the Years 1831, 1832, and 1833. Memoirs of the Royal Astron. Soc., Vol. VIII.

1834. Letter to F. BAILY, Esq. Notices of Astron. Soc., Vol. III. p. 75.

1834. Meteorological Observations made at Feldhausen, C. G. H., Dec. 21st, 1834. Athenæum.

1835. Annual Report of the South African and Scientific Institution. Read July 11th, 1835.

1835. Instructions for Making and Registering Meteorological Observations in South Africa and other Countries in the South Seas. (Published in London by M. ROBERTSON in 1835.)

1835. Account of Proceedings at the Cape of Good Hope. SCHUMACHER's Astron. Nachr. No. 281.

1835. Meteorological Observations made at Feldhausen (Athenæum), March 21st, 1835.

1835. " " " " " June 21st, 1835.

1835. " " " " " Dec. 21st, 1835,

1836. Extract of a Letter to C. LYELL, Esq., dated Feb. 20th, 1836, "On the primary Cause of Geological Upheavals and Disturbances." Printed in C. BABBAGE's Ninth Bridgewater Treatise, p. 202.

1836. A Letter to F. BAILY, Esq., giving an Account of the present Appearance of γ Virginis. Notices of Astron. Soc, Vol. III. p. 197.

1836. Meteorological Observations made at Feldhausen, March 21st, 1836 (Athenæum).

1836. " " " " June 21st, 1836 "

1836. " " " " Sept. 21st, 1836 "

1836. " " " " Dec. 21st, 1836 "

1836. Report of the Meteorological Committee of the South African Literary and Philosophical Institution (published by the Institution).

1836. A Letter to Dr. ADAMSON of the South African College.

1836. Extract of a Letter to R. I. MURCHISON, Esq., on the *Primum Mobile* of Geological Disturbances. Printed by C. BABBAGE in his "Ninth Bridgewater Treatise," p. 214.

1836. On the Cape of Good Hope Government Schools. Printed in Mr. C. BAMBURY's book on South Africa.

1836. Sixth Catalogue of Double Stars, observed at Slough, in the Years 1831 and 1832, with the Twenty-feet Reflector: containing the Places, Descriptions, and Measured Angles of Position of 286 of those Objects of which 105 have not been previously described. Reduced to the Epoch 1830. Memoirs Royal Astron. Soc., Vol. IX.

1837. A Letter to F. BAILY, Esq., dated Oct. 18th, 1836, on the Observations of the Comet of HALLEY, after the Perihelion Passage in 1836: made at Feldhausen, Cape of Good Hope. Read Jan. 13th, 1837. Trans. Royal Astron Soc.

1837. Meteorological Observations at Feldhausen in March 21st, 1837 (Athenæum).

1837. " " " June 21st, 1837 "

1837. " " " Sept. 21st, 1837 "

1837. " " " Dec. 21st, 1837 "

1837. Report of the Meteorological Committee. Read July 17th, 1837.

1838. Astron. Nachr., No. 354. Extract of a letter to Dr. MÄDLER on the sudden Change of Magnitude of the star η Argus

1838. Report of British Association, 1838. "Notices of Observations of Stars and Nebulæ observed at the Cape of Good Hope."

1838. Notice on HALLEY's Comet. Report of British Association, 1838.

1838. On a Method of Obtaining a great Increase of Light from a common Argand Burner. London, Edinburgh, and Dublin Phil. Magazine.

1839. Astron. Nachr., No. 372. Letter to Professor SCHUMACHER on the Variable Stars η Argus, α Hydræ, and α Cassiopeiæ; and on the comparative brightness of the Stars.

1839. Note on the Art of Photography; or on the Application of the Chemical Rays of Light to the purposes of Pictorial Representation. Notices of the Proceedings of the Royal Society of London. No. 37, p. 131.

1839. On the Variability of the Star α Cassiopeiæ. Notices of Royal Astron. Soc. May 10th, 1839.

1839. A Letter to the President of the British Association, giving an Account of the Production of analogous Colors by the Action of the colored prismatic Rays on Argentine Papers; and on a peculiar Property of the extreme red Rays. (Report of Proceedings of the British Association, 1839. Also Athenæum, No. 641.)

1839. Astron. Nachr., No. 405. "Requium of the Old Telescope," with a Translation into German by Madame MADLER.

1840. Address to the Astron. Soc. on occasion of delivery of a Medal to M. PLANA. (Notices of Astron. Soc., Feb. 1840.)

1840. On the Chemical Action of the Rays of the Solar Spectrum on Preparations of Silver and other Substances, metallic and non-metallic. Phil. Trans. Royal Soc. London, 1840.

1840. Report on the Subject of Terrestrial Magnetism in a Series of Resolutions, adopted by the British Association, at their Meeting in August, 1838, at Newcastle. Report of the British Association for the Advancement of Science for 1839.

1840. On the Variability and Periodical Nature of the Star α Orionis. Read Jan. 10th, 1840. Trans. Royal Astron. Soc.

1840. Review of GAUSS and W. WEBER'S allgemeine theorie des Erdmagnet; and of GAUSS'S Intensitas vis Magneticæ. Quarterly Review, No. 131. June, 1840.

1840. Report of the Committee of Physics, including Meteorology. Royal Soc. London.

1841. A Letter to Professor STEVELLY, Sec. British Association, "On Processes for producing Colored Photographs." British Association, 1841.

1841. On Sir WM. HAMILTON'S Observations of the Meteors of Aug. 9th, during the great Eruption of Vesuvius described by him. Athenæum, No. 721. Aug. 21st, 1841.

1841. An Address delivered on the Presentation of the Gold Medal of the Royal Astronomical Society to Professor BESSEL, at the Anniversary Meeting, Feb. 12th, 1841, for his Observations and Researches on the Parallax of 61 Cygni.

1841. On the Advantages to be attained by a Revision and Re-arrangement of the Constellations, with especial Reference to those of the Southern Hemisphere; and on the Principles upon which such Re-arrangement ought to be conducted. Memoirs of the Royal Astron. Soc. Vol. XII.

1841. Report of the Magnetic Committee of the British Association.

1841. Astron. Nachr., No. 442, On the Moon's Influence on the Weather.

1841. Remarks on certain Suggestions offered to the Commission for the Restoration of the lost Standard of Weights and Measures. Report of the Commission, 1841.

1841. Remarks on the Nomenclature of Coins, Weights, and Measures, and on some other Points connected with the Subject of the Restoration of the lost Standards. Report of the Commission.

1841. A Review of WHEWELL'S History and Philosophy of the Inductive Sciences. Quarterly Review, No. CXXXV.

1842. On the Action of the Rays of the Solar Spectrum on Vegetable Colors; and on some new Photographic Processes. Read June 16th, 1842 Phil. Trans., Part II., for 1842.

1842. The Walk. Translated in the original Metre from the German of F. SCHILLER. For private circulation.

1842. Extracts from a letter addressed by Sir J. F. W HERSCHEL to Mr. MURCHISON, explanatory of the Phenomena of the Freezing Caves of Ilitzkaia Zatcheta. Notices of the Geological Soc. of London. Vol. ? p. 697.

1842. On some Phenomena observed on Glaciers, and on the Internal Temperature of large Masses of Ice or Snow, with some Remarks on the natural Ice Caves which occur below the Limit of Perpetual Snow. Notices of the Geological Soc. Vol. p. 699.

1843. An Account of the great Explosion for the Removal of the Round Down Cliff at Dover, in a Letter to the Editor of the Athenæum. Ath., Feb. 1843.

1843. First Notice of the Great Comet of 1843. Times Newspaper, March 20th or 21st, 1843.

1843. A second Letter to the Editor of the "Times" on do. do., March, 1843.

1843. A Letter to the Editor of the Morning Herald on do. do., March 25th or 26th, 1843

1843. Letter to the Editor of the "Times" on the Comet and the Zodiacal Light, April, 1843.

1843. Account of an Aurora Borealis. Athenæum, No. 811, p. 465.

1843. Notice of a remarkable Photographic Process by which Dormant Pictures are rendered susceptible of Development by the Breath, or by keeping in a moist Atmosphere. Proceedings of the British Association for August, 1843.

1843. On the further Increase of η Argus. Notices of the Royal Astron. Soc., Nov 10th, 1843.

1843. Report of the Committee consisting of Sir JOHN HERSCHEL, the MASTER OF TRINITY COLLEGE, Cambridge, the DEAN OF ELY, Dr. LLOYD, and Colonel SABINE, appointed to conduct the Co-operation of the British Association in the System of Simultaneous Magnetical and Meteorological Observations. Report of the British Association for the Advancement of Science for 1843.

1843. A letter to the Editor of the Philosophical Magazine and Journal, dated Jan. 11th, 1843, On the Action of the Rays of the Solar Spectrum on the Daguerreotype Plate. From the London, Edinburgh, and Dublin Phil. Magazine and Journal of Science for Feb., 1843.

1843. Report of the Committee appointed for the Reduction of Meteorological Observations. From the Report of the British Association for the Advancement of Science for 1843.

1843. A Letter addressed to S. HUNTER CHRISTIE, Esq., Sec. Royal Soc., dated Nov. 15, 1842, on

certain Improvements on Photographic Processes and on Parathermic Rays of the Solar Spectrum. Phil. Trans., Part I. for 1843.

1844. Astron. Nachr., No. 493. Notice of Mr. CLERIHEWS's Observations of the Bifurcation of the tail of the Great Comet of 1843.

1844. Further Remarks on the Revision of the Constellations. Notices of Royal Astron. Soc., Vol. VI. p. 60.

1844. Contributions to Actinometry; also On the Amphitype, a new Photographic Process. British Association Report for 1844.

1844. Astron. Nachr., No. 520. "On M. HOUZEAU's Speculations on the Effect of Aberration in altering the Apparent Periods and Orbits of Double Stars."

1845. Address to the British Association on its Opening at Cambridge on June 19th, 1845. Report of British Association for 1845.

1845. On a Case of Superficial Color presented by a homogeneous Liquid internally colorless. Read Feb. 13th, 1845. Phil. Trans., Part I.

1845. On the Epipôlic Dispersion of Light, being a Supplement to a Paper entitled "On a Case of Superficial Color presented by a homogeneous Liquid internally colorless." Read April 3d, 1845. — Phil. Trans., Part I

1845. Sixth Report of the Committee, consisting of Sir J. HERSCHEL, the MASTER OF TRINITY COLLEGE, Cambridge, the DEAN OF ELY, Dr. LLOYD, and Colonel SABINE, appointed to conduct the Co-operation of the British Association in the System of Simultaneous Magnetical and Meteorological Observations. Report of the British Association for the Advancement of Science for 1844.

1845. Memoir of FRANCIS BAILY, Esq. From the Monthly Notices of the Royal Astronomical Society, Vol. VI, November, 1844.

1845. Report of a Committee, consisting of Sir JOHN HERSCHEL, Mr. WHEWELL, and Mr. BAILY (deceased), appointed by the British Association in 1840, for revising the Nomenclature of the Stars. Report of the British Association for the advancement of Science for 1844.

1845. "On a Seed-depositing Dibble," in a letter to the Editor of the Gardener's Chronicle. G. C. 1845, No. 12.

1846. Astron. Nachr., No. 584. Observations of BIELA's double Comet on Jan. 28th, 1846.

1846. Notices of Astron. Soc., Vol. VII. p. 95. Letter to Captain SMYTHE on the change of Magnitude of β Ursæ Minoris.

1846. Letter to Editor on LEVERRIER'S (ADAM'S) Planet. Athenæum, No. 988, p. 1019.

1846. Letter to Editor of the Guardian on do. do. Guardian, Oct. 28th, 1846.

1846. Letter to Editor of the Times, Oct., 1846.

1847. Obituary Notice of BESSEL. Notices of Astron. Soc., Feb. 12th, 1847.

1847. On the Action of Lime in Preserving liquid Manure. Gardener's Chronicle, 1847, No. 15, p. 244.

1847. Results of Astronomical Observations made during the Years 1834, 5, 6, 7, 8, at the Cape of Good Hope, being the Completion of a Telescopic Survey of the whole Surface of the visible Heavens, commenced in 1825. SMITH & ELDER, London.

1847. Astron. Nachr., No. 616. Extract of a Letter to Prof. SCHUMACHER on the Nomenclature of the small Planets.

1847. Letter to Editor of the Times on the Planet Neptune.

1847. Review of HUMBOLDT's Kosmos. Edinburgh Review, Jan. 1848.

1848. Obituary Notice of Miss CAROLINE LUCRETIA HERSCHEL. Athenæum, No. 1066.

1848. Address to the Royal Astron. Soc. on delivering Testimonials of Merit to Messrs. HANSEN, HENCKE, HIND, BISHOP, LUBBOCK, LEVERRIER, ADAMS, ARGELANDER, AIRY, EVEREST, and WEISSE. Notices, Feb., 1848.

1848. Athenæum, No. 1100, p. 1176. Letter to Editor on the Planet Neptune.

1848. Athenæum, No. 1100. On dividing the Vanishing Lines in Perspective.

1848. Athenæum, No. 1099, p. 1149. Account of a Lunar Rainbow.

1848. On the Effects of the Moon on the Weather. (Lord LOVELACE's Essay on Climate as connected with Husbandry.)

1849. Address delivered on presenting the Honorary Medal of the Royal Astronomical Society to WILLIAM LASSELL, Esq., of Liverpool. Royal Astron. Soc., Vol. IX. No. 4.

1849. On the Determination of the most probable Orbit of a Binary Star. Read April 13th, 1849. Trans. Royal Astron. Soc.

1849. Outlines of Astronomy. LONGMAN. London. (This has been translated into Arabic and Chinese.) N. B. — A 5th Edition much enlarged, published in 1858.

1849. Preface, Article Meteorology, Appendix to Astronomy, and Appendix to Ethnology in the "Naval Officer's Scientific Manual," edited by Sir J. HERSCHEL.

1850. Athenæum for Sat., Feb. 2d, 1850. Latin Epigram — "In cupidinem flantem." Also, Notice of an Error of 2 days left uncorrected in the Gregorian Reformation of the Calendar.

1850. Review of M. QUETELET's Lettres sur la Théorie des Probabilities, and Translation. Edinburgh Review, July, 1850.

1850. On the Algebraic Expression of the Number of Partitions of which a given Number is susceptible. Read May 16th, 1850. Phil. Trans., Part II., for 1850.
1851. Reclamation on Mr KEMP's claiming my Process for Separation of Iron from other Metals. Phil. Magazine, No. 170.
1853. On the Substitution of Bromine for Iodine in Photography. Journal of the Photographic Soc., No. 6, p. 70, June 21st, 1853.
1853. Athenæum, No. 1341, p. 83 Letter to Editor on Mr J. STEWART's new Photographic Processes.
1854. Evidence on Examination before the Select Committee of the House of Commons on the Decimal Coinage. See Report of Committee, 1853, p. 45–62.
1854. Circular issued to Bankers by Sir J. H., Master of the Mint, proposing Inquiries respecting the Circulating of the Florin. Report of the Committee for Decimal Coinage, 1857, p. 125.
1854. Analysis of Replies of Bankers, &c., to the Queries on the Circulation of the Florin. Preliminary Report of Committee on Decimal Coinage, 1857.
1856. Remarks on Slaty Cleavage and the Contorsian of Rocks, in a Letter to T. TYNDALE, Esq., F. R. S.
1856. Letter to Dr. BOOTH, Chairman to the Society of Arts, &c., on the proposed Examination System of the Society. Literarium and Educational Gazette, March 26th, 1856.
1857. The article "Meteorology," Encyclopædia Britannica, Vol. XIV.
1857. Letter in the "Times" signed "Helioscopus," April, 1857.
1857. A volume entitled "Essays from the Edinburgh and Quarterly Reviews with Addresses and other Pieces." LONGMAN & Co., London, one vol. 8vo.
1857. Replies to Lord OVERSTONE's Queries on the Decimal Coinage. Printed in the Report of the Committee for considering the Subject of the Decimal Coinage, 1857.
1858. Sensorial Vision: a Paper read before the Philosophical and Literary Society of Leeds, Sept. 30th, 1858.
1858. Introductory Address to the Chemical Section of the British Association on taking the Chair of that Section in Leeds, in 1858. Sept. 23d.
1858. Speech in the Town Hall at Leeds, on Occasion of presenting the Prizes for the Oxford Competitive Examination to the Successful Candidates. Reported in the Leeds Mercury, Sept. 1858.
1858. Report of the Joint Committee of the Royal Soc., and the British Association for procuring Continuance of the Magnetic and Meteorological Observations.
1858. Speech in the Leeds Commercial Exchange, on the Occasion of presenting Prizes to the Successful Candidates of the Young Men's Christian Institute for the Society of Arts Competitive Examinations.
1858. A Letter to the Editor of the Photographic News, dated Oct. 31st, 1858. Photographic News.
1858. A Letter to the Editor of the Photographic News, dated Nov. 6th, 1858, on the "Stereoscopic Angle."
1858. Obituary Notice of the late Very Rev. Dr. PEACOCK, DEAN OF ELY. From the Proceedings of the Royal Soc. for Nov 30, 1858.
1859. Remarks on Color Blindness. From the Proceedings of the Royal Soc. for May 26th, 1859.
1859. Manual of Meteorology. Extracted from the Admiralty Manual of Scientific Enquiry. Third Edition, 1859.
1859. A Letter to the Chairman of the Musical Pitch Committee on Uniform Musical Pitch, dated June 14th, 1859.
1859. The article "Physical Geography," Encyclopædia Britannica, Vol. XVII.
1859. A Letter to the Editor of the Photographic News, dated August 20th, on the Action of the Solar Spectrum upon certain Compounds of Silver. Photographic News.
1859. On a New Projection of the Sphere. Communicated to the Royal Geographical Society, March, 1859. Read April, 1859. Transactions of R. G. S., 1860–61.
1859. A Letter to the Chairman of the Society of Arts Committee for considering and reporting on the Adoption of a Standard of Musical Pitch. Journal of the Society of Arts, No. 346, Vol. VII., July 8th, 1859.
1859. A third Letter to the Editor of the Photographic News, dated July 16, 17, on the Action of the Solar Spectrum on Iodized, Bromized, and other Papers. Photographic News, July 17th, 1859.
1859. A fourth Letter, dated Aug. 20th, to the Editor of the Photographic News, on the Use of Lactate of Silver, and on the Spectra impressed on Iodinetted, Brominetted, and other Papers, by the Action of the Solar Rays. Photographic News, Sept 9th, 1859, Vol. III. p. 2.
1860. Athenæum, No. 1696, p. 581. British Modular Standard of Length. — Letter to Editor.
1860. Demonstration of Dr. BRINKLEY's Expression of the Coefficients in the Development of an Exponential Function Phil. Trans. Royal Soc. London, 1860.
1860. Athenæum, No. 1697, p. 216. A second Letter to the Editor of Athenæum on the Standard of Length.
1860. A fifth letter to Editor of Photographic News on Representation of Scenes in Action, and On Instantaneous Photography.

1860. Replies to Queries circulated by the Royal Commission for Light-House Buoys and Beacons, printed in the Report of the Commission.
1860. A Letter to the Editor of the "Photographic News," dated June 30th, 1860.
1861. An Article on a Swing Cot for Sea Voyages. From the Journal of the Society of Arts, Jan. 4th, 1861.
1861. The article "Telescope," Encyclopædia Britannica, Vol XXI.
——? Notice of a remarkable Occurrence of Serpentine at the Junction of Sienite with the Dolomite of the Tyrol.

This Catalogue of Titles was prepared by the author at our request. It would have been very difficult, if not impossible, for any other hand to have made it so complete. It is an interesting and valuable record of the incessant labor of half a century: remarkable both for the number of papers included and the variety of subjects treated. We had intended to give in this connection a short biographical sketch of the distinguished author, but want of space forbids. We will, however, refer our readers to a *Biographical Dictionary*, now in course of publication by Messrs. Griffin of London, for a sketch, which is, so far as it goes, exact, and has received Sir John Herschel's sanction.

While expressing our sincere thanks for the preparation of this catalogue and for a large number of the memoirs included in it, we would also acknowledge the receipt of the following very valuable papers by the author's illustrious father, Sir William Herschel: —

1789. Catalogue of a Second Thousand of New Nebulæ and Clusters of Stars; with a few Introductory Remarks on the Construction of the Heavens. Phil. Trans. Read at the Royal Society, June 11th, 1789.

1796. On the Method of observing the Changes that happen to the Fixed Stars; with some Remarks on the Stability of the Light of our Sun. To which is added a Catalogue of Comparative Brightness, for ascertaining the Permanency of the Lustre of Stars. Read before the Royal Society, Feb. 25th, 1796. Phil. Trans.

1796. On the Periodical Star α Herculis; with Remarks tending to establish the Rotatory Motion of the Stars on their Axes. To which is added a Second Catalogue of the Comparative Brightness of the Stars. Read before the Royal Society, June 9th, 1796. Phil. Trans.

1797. A Third Catalogue of the Comparative Brightness of the Stars; with an Introductory Account of an Index to Mr. Flamsteed's Observations on the Fixed Stars, contained in the Second Volume of the Historia Cœlestis. To which are added several useful Results derived from that Index. Read before the Royal Society, May 18th. Phil. Trans.

1799. A Fourth Catalogue of the Comparative Brightness of the Stars. Read before the Royal Society, February 21st. Phil. Trans.

1802. Catalogue of Five Hundred New Nebulæ, Nebulous Stars, Planetary Nebulæ, and Clusters of Stars; with Remarks on the Construction of the Heavens. Read before the Royal Society, July 1st, 1802. Phil. Trans.

1803. Account of the Changes that have happened, during the last twenty-five years, in the Relative Situation of Double Stars; and an Investigation of the Cause to which they are owing. Read before the Royal Society, June 9th. Phil. Trans.

Editorial Items.

Sir John Herschel's Portrait. — It gives us sincere pleasure to be able to present to our readers an excellent likeness of Sir John Herschel. It was engraved by the distinguished artist, H. Wright Smith, Esq. of Boston, from a Photograph in the possession of Miss Maria Mitchell, of Nantucket, who says the likeness is as good as it can be. Miss Mitchell has kindly allowed us the perusal of an extract of a letter from Lady Herschel, in which a decided preference is given to this Photograph.

We can assure our friends that the artist has been remarkably successful in reproducing the likeness; and it is with the deepest satisfaction that we present it to the numerous admirers of the distinguished original on this side of the Atlantic, by whom we feel assured it will be received with the warmest welcome.

The following gentlemen have sent solutions of the problems in the December number: —

H. J. McMurphy, Londonderry, N. H.; Asher B. Evans, Madison University, Hamilton, N. Y.; W. L. Marcy, Coventry, N. Y.; J. M. Terrill, Linneus, Me.; G. B. Hicks, Cleveland, Ohio; Edwin A. Dodds, Gouverneur, N. Y.; William Timpson, White Plains, N. Y.; James Clark, Wayne, Maine; E. S. Stearns, Chester, N. J.; David Trowbridge, Perry City, N. Y., W. W. Lemmon, M'Kendree College, Lebanon, Ill.; W. G. N., Philadelphia, Pa.; H. P. Henderson, N. Y. City.

Prizes for Solutions. — Those entitled to these Prizes will please address Messrs. Sever & Francis, stating the number of the Monthly in which the prize is awarded, its amount, the books desired, and how they should be sent. It must not be forgotten that the postage on books must be *prepaid.*

It would add very much to the interest and value of this department of the Monthly if the competition could be largely increased. It is not certain that the good accomplished at present warrants the expense, labor, and space in the Monthly devoted to these prizes.

Books Received. — A Treatise on the Differential and Integral Calculus. By A. E. Church, LL. D., Professor of Mathematics in the U. S. Military Academy, West Point. A new edition. Published by A. S. Barnes & Burr, 51 and 53 John Street, N. Y. (Notice in next number.)

An Elementary Treatise on the Dynamics of a System of Rigid Bodies. With numerous Examples. By Edward John Routh, M. A., Fellow and Assistant Tutor of St. Peter's College, Cambridge, Examiner in the University of London. Cambridge: Macmillan & Co.; and 23 Henrietta Street, Covent Garden, London. 1860.

Worcester's Quarto Dictionary The Standard

VERY SIGNIFICANT FACTS.

The following recommendations are from some of the most distinguished American and English scholars. They are but a few from many which have been received, testifying to the superiority of Worcester's Quarto Dictionary. These testimonials are of the highest value, for they have all been given during the present year, and after an examination of this work and of that which is endeavoring to hold the position of a rival. The scholars of America and of England, with scarcely an exception, have decided in favor of Worcester. Not a single scholar, equal in authority to any one mentioned below, can be cited as giving, after a comparison of the two works, the preference to Webster's Dictionary. We give the testimony: —

From C. C. Felton, LL.D., *President of Harvard College.*

Aware of the labor and care which had been devoted to this (the department of scientific terms) as well as to other parts of the work, I felt assured that Worcester's Quarto Dictionary would more nearly meet the public wants than any other hitherto published.

My expectations have been more than fulfilled. I find it not only rich beyond example in its vocabulary, but carefully elaborate in all the details, and thoroughly trustworthy as a guide to the most correct and elegant usage of the language.

From the Rev. Joseph Bosworth, D.D., F.R.S., *Professor of Anglo-Saxon, Oxford, England.*

It is the most complete and practical, the very best, as well as the cheapest English Dictionary that I know.

From George P. Marsh, LL.D.

The work of Dr. Worcester is unquestionably much superior to any other general dictionary of the language in every one of these particulars (orthography, pronunciation, definition, fulness of vocabulary, and precision and distinctness of definition).

From Rev. W. Whewell, D.D., *Master of Trinity College, England.*

I have repeatedly consulted the Dictionary since it has been in my possession, and have seen reason to think it more complete and exact than its predecessors.

From Charles Richardson, LL.D., *the oldest living English Lexicographer, England.*

I sincerely hope you may enjoy from your brethren, both in America and England, that tribute of honor to which you have earned so undoubted a title.

From D. R. Goodwin, D.D., *President of Trinity College, Hartford.*

It was but a short time since that I was led to commend another dictionary as, on the whole, and with some exceptions, the best and most complete thing of the kind within my knowledge. The commendation was honestly given at the time; but now *it must be withdrawn in favor of yours.* I consider your dictionary, in orthography, pronunciation, and definitions, as superior to any of its predecessors.

From Rev. W. B. Sprague, D.D., *of Albany, N. Y.*

My opinion of Worcester's Quarto Dictionary, after having given it as extended an examination as my circumstances would admit, is, that there is no other dictionary in the language that compares with it for completeness, accuracy, comprehensiveness, and precision, and perhaps I ought to add, that I have arrived at this conclusion rather contrary to a preconceived opinion.

From Rev. Henry A. Boardman, D D., *of Philadelphia.*

I particularly like it (the Dictionary), 1. Because of its very comprehensive character; 2 Because it adheres to the *settled orthography* of our noble language, — discarding those innovations which, however countenanced by certain publishing-houses, have never to any extent been accepted by the scholars of our country.

From Louis Agassiz, LL.D.

It is of great importance, when the nomenclature of science is gradually creeping into common use, that an English lexicon should embrace as much of it as is consistent with the language we speak. I am truly surprised and highly delighted to find you have succeeded far beyond my expectations in making the proper selection, and combining with it a remarkable degree of accuracy. More could hardly be given except in a scientific cyclopædia.

The following lines are quoted from Harper's Magazine for September. They serve to show very truthfully the comparative value of recent and old commendations: —

"Injustice. — Our attention has been called to a species of injustice of which publishers are sometimes guilty, in publishing commendations of school-books, without giving the dates when they were written. Especially does this merit reproval when these commendations are old, and when it is known that the writers have subsequently commended other and later publications in the same department. It will readily be seen that this is frequently not only an act of injustice to teachers who have had the courtesy to commend a book, but that it is also a fraud upon the public."

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THE NORTH AMERICAN REVIEW. No. CLXXXIX. — For October, 1860.

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